Equation (4.5) for $w_1(x, y)$ with homogeneous boundary conditions and zero right-hand side yields a zero solution. Consequently

$$w = w(x) = \frac{d}{\tau_y}\left(b\frac{d\varphi_s}{dx} + \varphi_T\right),$$

$$T = T(x) = \frac{d}{\tau_y}\left(1 - \frac{1}{\tau_x^2}\frac{d^2}{dx^2}\right)\left(b\frac{d\varphi_s}{dx} + \varphi_T\right)$$

$$v = y\frac{d}{dx}\left[f - \varphi_s + \frac{1}{b\tau_y\tau_x^2}\frac{d}{dx}\left(b\frac{d\varphi_s}{dx} + \varphi_T\right)\right]$$

Unlike in Sect. 3 the radiation transfer affects the flow of gas in a channel with adiabatic wall even in the first approximation when $\tau_x \sim 1$. When $\tau_x \to \infty$ all formulas in Sect. 4 coincide with corresponding formulas in Sect. 3.

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ON NONLINEAR DAMPING OF PLANE ACOUSTIC PULSES IN A RADIATIVE GAS

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The nonlinear evolution of small amplitude waves in a viscous heat-conducting gas at low and high Boltzmann radiation number is investigated on the example of the piston problem.

Results of calculations of the formation of weak stationary shock waves and of the damping of triangular compression pulses are presented.

Radiant energy transport in small amplitude waves was considered in numerous publications (see, e.g., the bibliography in [1, 2]). The majority of investigations of radiative gas acoustics were carried out in linear approximation. Linear analysis shows that in a radiative gas the propagation and damping velocities of small perturbations depend on the charactistic optical thickness τ and on the Boltzmann radiation number N_{Bo} which defines the ratio of convection and radiant energy fluxes.

When $N_{Bo} \sim 1$ and $\tau \sim 1$, the perturbation propagation velocity lies between the isothermal a_T and the isentropic a_s speeds of sound. The basic process which leads to the damping of perturbations is in this case the dissipation of radiant energy by the perturbed region. Since damping takes a comparatively short time, the nonlinear convection effects, which are cumulative and depend on the input equations of gasdynamics, cannot make themselves felt. Thus for $N_{Bo} \sim 1$ and $\tau \sim 1$ the equations of linear acoustics of radiative gas completely define the damping process.

When $N_{Bo} \gg 1$ and, also, when $\tau \gg 1$ and $\tau \ll 1$, velocity a is close to a_s while when $N_{Bo} \ll 1$ and $\tau \sim 1$, velocity a is close to a_T. In these limit cases the perturbation damping by radiant energy dissipation is small, and the nonlinear convection effects are evident to their full extent.

Equation of the theory of nonlinear acoustics for waves in a radio-active gas were obtained in [2]. Laws of nonlinear damping of optically thick and thin perturbations were investigated in [3]. It should be noted that radiative transport of energy can strongly affect the damping of perturbations even in a cold gas. For example, according to estimates by the linear theory damping of low-frequency acoustic waves at sea level of the Earth's atmosphere is basically due to energy transport in infra-red bands of steam absorption [4, 5]. It is also known (*) that subsonic waves propagate in the atmosphere of planets for considerable distances. Hence nonlinear effects can considerably affect the law of their damping. It is interesting to investigate the combined effect of radiative transport and nonlinear convection effects.

1. Let us consider a half-space filled with a viscous heat-conducting radiative gas and bounded by a plane wall (piston) moving according to the law

$$x_w(t) = \varepsilon L f(a_0 t / L), \; u_w(t) = \varepsilon a_0 f'(a_0 t / L), \; \varepsilon \ll 1 \tag{1.1}$$

where, and everywhere below, primes denote derivatives of the dimensionless function f, which defines the law of piston motion, with respect to its argument; x_w defines the piston position in a system of coordinates whose x-axis is normal to its surface; t is the time; L is a characteristic length of piston displacement; ε is a small dimensionless parameter; a_0 the characteristic propagation velocity of small perturbations, and u_w is the piston velocity. It is convenient to assume in what follows that a_0 coincides with either the isentropic or the isothermal speed of sound.

(*) See, e.g., Golitsyn, G. S. and Chunchuzov, E. P., Acoustic-gravitational waves in the atmosphere. Survey of Experimental and theoretical data. VINITI, dep. No. 7200-73 (RZhGeofiz., No. 3, 3A-144, 1974)

If the assumptions usually made in dynamics of radiative gas about local thermodynamic equilibrium, absence of dissipation, and negligibly small pressure and internal energy of radiation in comparison with the pressure and internal energy of gas, are applied in this case, the motion of gas is defined by equations [6]

$$\frac{\partial\rho}{\partial t}+\frac{\partial\rho u}{\partial x}=0,\quad \rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)+\frac{\partial p}{\partial x}=\frac{d}{\partial x}\zeta\frac{\partial u}{\partial x} \tag{1.2}$$

$$\rho T\left(\frac{\partial s}{\partial t}+u\frac{\partial s}{\partial x}\right)=\zeta\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}-\frac{\partial q}{\partial x}$$

$$\mu\frac{\partial I_\nu}{\partial x}=\varkappa_\nu(B_\nu-I_\nu),\quad q=\int_0^\infty 2\pi\int_{-1}^{1}\mu I_\nu\,d\mu\,d\nu$$

where s is the specific entropy of gas; ζ is the coefficient of longitudinal viscosity; λ is the coefficient of thermal conductivity; q is the total flux of radiant energy; ν is the radiation frequency; μ is the cosine of the angle between the x-axis and the light ray; I_ν is the radiation intensity; $\varkappa_\nu$ is the coefficient of absorption of gas, and B_ν is the Planck function that specifies the intensity of equilibrium radiation.

System (1.2) must be supplemented by two relationships (equations of state) which link the thermodynamic variables ρ, p, T, and s and specify the dependence of coefficients of viscosity ζ, thermal conductivity λ and of absorption $\varkappa_\nu$ on the two of these.

The boundary condition for Eqs. (1.2) which defines the absence of mass flow through the piston surface is of the form

$$u\,[t,\ x_w\,(t)]=u_w\,(t) \tag{1.3}$$

The second boundary condition which defines the radiative transport of energy at the piston surface will be formulated when required.

In what follows it is convenient to use two corollaries of Eqs. (1.2) [2]

$$\frac{\partial p}{\partial t}+(u+a_s)\frac{\partial p}{\partial x}+\rho a_s\left[\frac{\partial u}{\partial t}+(u+a_s)\frac{\partial u}{\partial x}\right]= \tag{1.4}$$

$$\frac{1}{\rho T}\left(\frac{\partial p}{\partial s}\right)_\rho\left[\zeta\left(\frac{\partial u}{\partial x}\right)^2+\frac{\partial}{\partial x}\lambda\frac{\partial T}{\partial x}-\frac{\partial q}{\partial x}\right]+a_s\frac{\partial}{\partial x}\zeta\frac{\partial u}{\partial x},$$

$$a_s=\left(\frac{\partial p}{\partial\rho}\right)_s^{1/2}$$

$$a_T\left[\frac{\partial\rho}{\partial t}+(u+a_T)\frac{\partial\rho}{\partial x}\right]+\rho\left[\frac{\partial u}{\partial t}+(u+a_T)\frac{\partial u}{\partial x}\right]= \tag{1.5}$$

$$\frac{\partial}{\partial x}\zeta\frac{\partial u}{\partial x}-\left(\frac{\partial p}{\partial T}\right)_\rho\frac{\partial T}{\partial x},\quad a_T=\left(\frac{\partial p}{\partial\rho}\right)_T^{1/2}$$

Let us define the Reynolds number N_{Re} and the Prandtl and Boltzmann numbers N_{Pr} and N_{Bo} respectively, in terms of parameters of the unperturbed state of gas which will be everywhere denoted by subscript zero.

We have

$$N_{\mathrm{Re}}=\rho_0a_0L\,/\,\zeta_0,\ N_{\mathrm{Pr}}=C_{p0}\zeta_0\,/\,\lambda_0,\ N_{\mathrm{Bo}}=\rho_0a_0^3\,/\,(\sigma T_0^4) \tag{1.6}$$

where C_{p0} and C_V are specific heats of gas at constant pressure and constant volume, respectively, and $\sigma=2\pi^5k^4\,/\,(15h^3c^2)$ is the Stefan-Boltzmann constant.

Two limit modes $N_{\mathrm{Bo}}\gg 1$ and $N_{\mathrm{Bo}}\ll 1$ in which it is necessary to take into account the convective nonlinearity of flow are considered here. In the first case the perturbation spreads through a comparatively cold and dense gas. The second case relates to the spreading of perturbation through a fairly hot gas whose density is comparatively low. We also assume that

$$N_{\mathrm{Re}} \gg 1 \tag{1.7}$$

since $N_{\mathrm{Re}} \sim L/l_m$, where l_m is the length of gas particles mean free path.

To take into account the boundary conditions (1.3) it is necessary to use the dimensionless variables

$$t_1 = a_0 t / L, \qquad x_1 = x / L \tag{1.8}$$

It is reasonable to seek the solution in the form of asymptotic expansions [2]

$$u = \varepsilon a_0 u_1 + \ldots, \quad \rho = \rho_0 (1 + \varepsilon \rho_1 + \ldots) \tag{1.9}$$

$$p = p_0 + \varepsilon \rho_0 a_0^2 p_1 + \ldots, \quad s = s_0 (1 + \varepsilon_s s_1 + \ldots)$$

$$T = T_0 (1 + \varepsilon_T T_1 + \ldots), \quad q = \varepsilon_T \sigma T_0^4 q_1 + \ldots$$

$$I_\nu = B_{\nu 0} + \varepsilon_T T_0 \frac{\partial B_\nu}{\partial T_0} I_{\nu 1} + \ldots$$

Then from (1.1) and (1.3) for the perturbation of velocity u_1 we obtain the following boundary conditions [7]:

$$u_1 (t_1, 0) = u_w (t_1) \tag{1.10}$$

In expansions (1.9) ε_s and ε_T are small parameters whose relation with parameter ε, which defines the piston velocity, and with the Boltzmann number N_{Bo} depends on the considered mode ($N_{\mathrm{Bo}} \gg 1$ or $N_{\mathrm{Bo}} \ll 1$).

Using variables (1.8) we obtain for the perturbations linear equations which are valid when $t_1 \sim 1$ and the perturbation is fairly close to the piston (in the closest zone). After a reasonably long time ($t_1 \gg 1$) when the perturbation is far away from the piston (in the distant zone), these equations are no longer valid, even if only because of the singularity in second approximation equations of linear acoustics [7]. To correctly define the evolution of perturbations when $t_1 \gg 1$ it is necessary to use variables

$$t_2 = \Delta a_0 t / L, \quad x_2 = (x - a_0 t) / L, \quad \Delta \ll 1 \tag{1.11}$$

which together with expansions (1.9) yield equations for nonlinear short waves [2]. The initial condition for these equations may be obtained by asymptotic joining with the linear solution that is valid at the initial instants of time [7].

2. Let us, first, consider the mode of considerable Boltzmann numbers

$$N_{\mathrm{Bo}} \gg 1 \tag{2.1}$$

when the velocity of perturbation propagation is close to the isentropic speed of sound. For this we set in (1.1), (1.6), (1.8), (1.9), and (1.11) $a_0 = a_{s0}$ and use the equation of state of the form $p = p(\rho, s)$ and $T = T(\rho, s)$.

Taking into account assumptions (1.7) and (2.1) and the boundary condition (1.10), we obtain from (1.2) and (1.9) in the closest zone and $t_1 \sim 1$ the perturbations of gasdynamic quantities and determine the order of small parameters ε_T and ε_s

$$u = \rho = p = T = f'(t_1 - x_1), \quad \frac{\partial s}{\partial t_1} = \frac{N_{\mathrm{Bo}}}{N_{\mathrm{Re}} N_{\mathrm{Pr}}} \frac{C_{p0} T_0}{a_{s0}^2} \frac{\partial^2 T}{\partial x_1^2} - \frac{\partial q}{\partial x_1} \tag{2.2}$$

$$\varepsilon_T = \varepsilon \left(\frac{\partial \ln T}{\partial \ln \rho_0} \right)_s \sim \varepsilon, \quad \varepsilon_s = \frac{\varepsilon}{N_{\mathrm{Bo}}} \frac{a_{s0}^2}{T_0 s_0} \ll \varepsilon \tag{2.3}$$

where subscripts at perturbations are henceforth omitted.

Thus in the $N_{\mathrm{Bo}} \gg 1$ mode entropy perturbations are considerably smaller than the perturbations of remaining gasdynamic quantities. Such flows were called in [2] quasi-isentropic (isentropic in the first approximation). In the first approximation the radiative transport of energy affects the flow dynamics in the closest zone, while solution (2. 2) defines a perturbation that propagates at the isentropic speed of sound without altering its form. The radiation intensity, radiative heat flux, and the perturbation of entropy are determined after integration of the transport equation in which the dependence of the absorption coefficient $\varkappa_\nu$ and of the Planck function B_ν on coordinate x_1 and time t_1 were previously determined with the use of (2. 2).

In the distant zone $t_2 \sim 1$ and $x_2 \sim 1$ assumptions (1. 7) and (2. 1) also lead to links (2. 3) between the small parameters and to the following integrals for perturbations [2]:

$$u = \rho = p = T \tag{2.4}$$

In variables (1. 11) of the distant zone coordinate $x_{w2} = \varepsilon f(t_2 / \Delta) - t_2 / \Delta$, $\Delta \ll 1$ corresponds to the piston. Hence it is possible to assume in the first approximation that in the distant zone $x_{w2} = -\infty$ and to consider the transport of radiant energy in a boundless space. In that case expansions (1. 9) yield the following formula for the derivative of radiant energy flux [2] :

$$\frac{\partial q}{\partial x_2} = 16\tau_0 (T - w), \quad \tau_0 = L\varkappa_0, \quad \varkappa_0 = \int_0^\infty \varkappa_{\nu 0} \frac{\partial B_\nu}{\partial T_0} d\nu \Bigg/ \int_0^\infty \frac{\partial B_\nu}{\partial T_0} d\nu \tag{2.5}$$

where $\varkappa_0$ is the acoustic absorption coefficient and τ_0 is the acoustic optical thickness. The quantity w is the perturbation of radiant energy density suitably averaged with respect to the radiation frequency. It is related to the temperature perturbation by the integral operator

$$w = \frac{\tau_0}{2} \int_{-\infty}^{\infty} T(t_2, \xi) F_1(\tau_0 | x_2 - \xi |) d\xi \tag{2.6}$$

$$F_1(y) = \int_0^\infty \varkappa_{\nu 0} \frac{\partial B_\nu}{\partial T_0} E_1\left(\frac{\varkappa_{\nu 0}}{\varkappa_0} y\right) d\nu \Bigg/ \left(\varkappa_0^2 \int_0^\infty \frac{\partial B_\nu}{\partial T_0} d\nu\right), \quad E_1(z) = \int_1^\infty \frac{e^{-sz}}{s} ds$$

We emphasize that formulas (2. 5) and (2. 6) are valid for any ratio $\varepsilon / \varepsilon_T$.

For simplicity we apply the approximate method proposed and analyzed in [8]. We approximate kernel F_1 of the integral operator (2. 6) by the exponent

$$F_1(y) \approx n \exp(-ny), \quad n = \text{const} > 0 \tag{2.7}$$

after which (2. 6) is reduced by double differentiation with respect to the coordinate to the differential equation

$$\partial^2 w / \partial x_2^2 = n^2 \tau_0^2 (w - T) \tag{2.8}$$

The optimum choice of the approximation constant n depends on the specific dependence of the absorption coefficient $\varkappa_{\nu 0}$ and frequency ν which will not be considered here.

Below it is convenient to use Eq. (1.4) [2]. Then with allowance for assumptions (1.7) and (2.1), expansions (1.9), relationships (2.3) between the small parameters, formula (2.5) for the derivative of radiant energy flux, Eq. (2.8), and integrals (2.4) in variables (1.11), we obtain the following equations:

$$\frac{\partial u}{\partial t_2} + u\frac{\partial u}{\partial x_2} = \frac{1}{N_s}\frac{\partial^2 u}{\partial x_2^2} + \frac{\tau}{b_s}(w - u), \quad \frac{\partial^2 w}{\partial x_2^2} = \tau^2 (w - u) \tag{2.9}$$

$$N_s = \frac{2N_{\mathrm{Re}}\varepsilon m_{s0}}{1 + (\gamma_0 - 1)/N_{\mathrm{Pr}}}, \quad b_s = \frac{N_{\mathrm{Bo}}\varepsilon n C_{p0} T_0 m_{s0}}{8 a_{s0}^2 (\gamma_0 - 1)}, \quad \tau = n\tau_0$$

$$m_{s0} = \frac{1}{2\rho_0^3 a_{s0}^2}\left(\frac{\partial^2 p}{\partial V_0^2}\right)_s, \quad V = \frac{1}{\rho}, \quad \gamma_0 = \frac{C_{p0}}{C_{V0}}, \quad \Delta = \varepsilon m_{s0}$$

which define the nonlinear propagation of acoustic perturbations in a radiative gas when $N_{\mathrm{Bo}} \gg 1$:

The parameter N_s in Eqs. (2.9) defines the effect of viscosity and corpuscular thermal conductivity. When $N_s \gg 1$ this effect is small.
The parameter b_s which is proportional to the Boltzmann number defines the effect of radiation. It is small when $b_s \gg 1$. The parameter τ is the effective optical thickness of perturbation.

The initial condition for Eqs. (2.9) is obtained by joining with solution (2.2) which is valid at the initial instants of time. Joining is carried out in the intermediate bounds [7]

$$x_* \equiv x_2 = x_1 - t_1, \quad t_* = \delta(\varepsilon) t_1$$
$$\delta(\varepsilon) \to 0, \quad \delta(\varepsilon)/\varepsilon \to \infty \quad \text{при } \varepsilon \to 0 \tag{2.10}$$

and yields the initial condition

$$u(0, x_2) = f'(-x_2) \tag{2.11}$$

Results of numerical computations of Eqs. (2.9) are shown in Figs. 1 and 2. To isolate the effect of radiant energy transport $N_s = 200$ was assumed in all computations, since at high values of parameter N_s the viscosity effect is essentially reduced to smoothing out weak and strong discontinuities. Allowance for low viscosity makes it possible to carry out continuous computation through shock waves that may appear in the solution when parameter b_s is fairly large. Computations were carried out by the two-sheeted implicit difference scheme. Terms which describe in (2.9) viscosity and radiation were taken from the upper time layer in order to avoid the difficulties associated with the instability of computation by the explicit scheme. Difference equations were solved by running through the matrices.

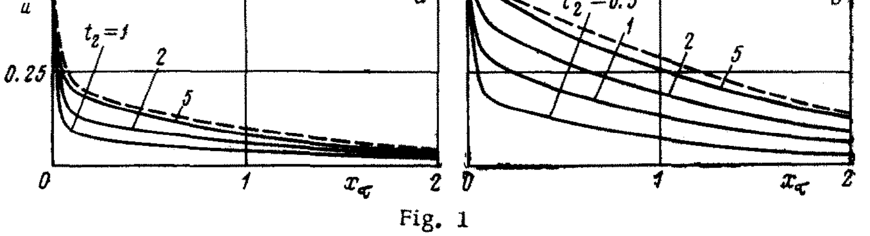

Fig. 1

Figure 1 shows the formation of a weak stationary shock wave induced by a piston which at the instant of time $t = 0$ begins to penetrate into the gas at low constant velocity In this case $f(t_1) = 0$ when $t_1 < 0$ and $f(t_1) = t_1$ when $t_1 \geqslant 0$. When the radiation transport is defined by the single-term exponential approximation model (2.8), the considered problem has only one characteristic linear dimension : the effective length of the radiation free path $l_0 = 1/(n\varkappa_0)$. Hence it is possible to set without loss of generality in (2.9) $\tau = 1$. In Fig. 1 the coordinate $x_\tau = x_2 - t_2/2$ at which the shock wave front is stationary is measured from the sound plane where $u = {}^1/_2$. In this case the solution of Eqs. (2.9) is symmetric about point $x_\tau = 0$, $u = {}^1/_2$, hence only the right-hand half of the flow is shown in the diagrams. The dash lines relate to stationary solutions toward which the perturbation evolves.

The compression shocks partly dispersed by radiation are clearly visible in Fig. When the effect of radiation is moderate (Fig. 1, a, where $b_s = 4$),) the compression shock remains also in the stationary solution. If the radiation effect is stronger (Fig. 1, b where $b_s = 1$), the compression shock is completely blurred by radiation, and the stationary wave is totally dispersed.

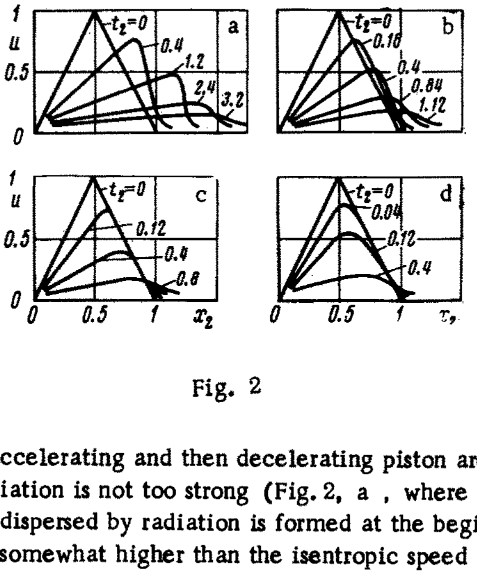

Fig. 2

If the stationary analog of Eqs. (2.9) is analyzed for $N_s = \infty$ in the neighborhood of the singular point which corresponds to the sound plane, it is possible, as in [9], to obtain the sufficient condition of existence of weak stationary partly dispersed shock waves $b_s > \sqrt{2}$. Numerical computations show that condition is also the necessary one, since for $b_s < \sqrt{2}$ a weak stationary wave is completely dispersed by radiation. The results of computation of damping of a triangular compression pulse of optical thickness $\tau = 1$ induced by a uniformly accelerating and then decelerating piston are shown in Fig. 2. When the effect of radiation is not too strong (Fig. 2, a, where $b_s = 2$), a compression shock weakly dispersed by radiation is formed at the beginning. It propagates at a velocity that is somewhat higher than the isentropic speed of sound. After some time the shock is completely blurred by radiation, and the acoustic pulse becomes an optically thick perturbation with mildly sloping forward and rear fronts. When the radiation effect is stronger (Fig. 2, b where $b_s = 0.6$), the incipient compression shock is strongly dispersed by radiation, which is shown by the difference in the slopes of shock fronts in Figs. 2, a and b. The case of $b_s = 0.4$ when convective nonlinear effects are balanced by radiation and the slope of the perturbation forward slope remains virtually unchanged is shown in Fig. 2, c. Finally, when the effect of radiation is strong (Fig. 2, d, where $b_s = 0.2$), the perturbation is rapidly damped and the effect of convection nonlinearity on the flow is almost negligible.

It is seen from Fig. 2 that at later instants of time the pulse becomes optically thick and its amplitude becomes small in comparison with the initial one.

Since we are interested in the law of nonlinear damping of the pulse when $t_2 \to \infty$, hence we introduce variables

$$t_3 = \Delta_t t_2, \quad x_3 = \Delta_x x_2, \quad \Delta_t \ll 1, \quad \Delta_x \ll 1$$

and expansions

$$u = \varepsilon_u u_3 + \ldots, \quad w = \varepsilon_w w_3 + \ldots, \quad \varepsilon_u \ll 1, \quad \varepsilon_w \ll 1$$

Substituting these expansions into (2. 9) we obtain

$$\frac{\partial u_3}{\partial t_3} + u_3 \frac{\partial u_3}{\partial x_3} = \left(\frac{1}{N_s} + \frac{1}{b_s \tau}\right) \frac{\partial^2 u_3}{\partial x_3^2}, \quad w_3 = u_3, \quad \Delta_x = \varepsilon_u = \varepsilon_w = \sqrt{\Delta_t}$$

A similar equation, often called the Burgers equation, defines the nonlinear propagation of small amplitude waves in a viscous gas [10]. Thus, when $t_2 \to \infty$ the nonlinear perturbation in a radiative gas is damped as the perturbation in a viscous gas with the effective parameter $N_s^* = N_s b_s \tau / (N_s + b_s \tau)$.

3. Let us consider the mode at small Boltzmann numbers

$$N_{\text{Bo}} \ll 1 \tag{3.1}$$

In the absence of external radiation sources at temperature that differs considerably from that of gas, then for $N_{\text{Bo}} \ll 1$ the powerful energy transport by radiation results in the equalization of temperatures in various flow regions. Hence the perturbation of temperature must be considerably smaller than the perturbations of remaining thermodynamic quantities so that

$$\varepsilon_T \ll \varepsilon \tag{3.2}$$

In [2] such flows are called quasi-isothermal.

If we assume in our problem that the piston radiates as an ideal black body temperature T_w, then the flow is quasi-isothermal when $|T_w - T_0| \sim \varepsilon_T T_0$. The other case of quasi-isothermal flow, which is considered below, is that of the adiabatic piston which completely or nearly completely reflects radiation.

When $N_{\text{Bo}} \ll 1$ the perturbation propagation velocity is close to the isothermal speed of sound. Hence we set $a_0 = a_{T0}$ in (1. 1), (1. 6), (1. 8), (1. 9), and (1. 11) and use the equation of state of the form $p = p(\rho, T)$ and $s = s(\rho, T)$. Then, taking into account assumptions (1. 7), (3. 1) and (3. 2) and boundary conditions (1. 10), in the near zone $t_1 \sim 1$ and $x_1 \sim 1$ for the perturbations of gasdynamic parameters we obtain

$$u = \rho = p = -s = f'(t_1 - x_1), \quad \frac{\partial s}{\partial t_1} = \frac{1}{16\tau_0} \frac{\partial q}{\partial x_1} \tag{3.3}$$

and determine the small parameters ε_s and ε_T

$$\varepsilon_s = -\varepsilon \left(\frac{\partial \ln s}{\partial \ln \rho_0}\right)_T \sim \varepsilon, \quad \varepsilon_T = -\frac{\varepsilon N_{\text{Bo}}}{16\tau_0} \left(\frac{\partial \ln \rho}{\partial \ln T_0}\right)_p \ll \varepsilon \tag{3.4}$$

Solution (3. 3) defines the perturbation which propagates at the isothermal speed of sound without changing its form.

Let us determine the temperature perturbation. In the first approximation the coordinate $x_1 = 0$ corresponds in the near zone $t_1 \sim 1$, $x_1 \sim 1$ to the piston. We assume, for simplicity, that the piston completely reflects radiation like a mirror in conformity with laws of geometrical optics. Then, owing to symmetry, the radiative transport of energy in region $x_1 > 0$ does not vary, if one formally considers the unbounded space in which the perturbation of gas temperature, continued into region $x_1 < 0$ is even : $T(t_1, x_1) = T(t_1, -x_1)$. Further we use expansions (1.9) which yield equations that coincide with (2.5) and (2.6) to within the accuracy of the substitution of t_1, x_1 for t_2 and x_2. We then use the exponential approximation (2.7) which yields an equation of the form (2.8) but with a derivative with respect to x_1 in its left-hand side. Finally, using formulas (3.3) we obtain the equations which determine temperature perturbations and the mean density perturbation of radiant energy

$$\partial^2 w / \partial x_1^2 = -\tau^2 f''(t_1 - x_1), \quad T = w + f''(t_1 - x_1), \quad \tau = n\tau_0 \qquad (3.5)$$

Since the piston is assumed to be adiabatic, hence at its surface $q = 0$, i.e. $\partial w / \partial x_1 = 0$ when $x_1 = 0$. Furthermore the radiant energy density w must be a continuous function. These conditions and (3.5) make it possible to determine the perturbation of temperature

$$T = f''(t_1 - x_1) - \tau^2 [f(t_1 - x_1) - f'(t_1)(t_1 - x_1)] \qquad (3.6)$$

At the piston surface $x_1 = 0$

$$T = f''(t_1) - \tau^2 [f(t_1) - f'(t_1)t_1] \qquad (3.7)$$

When the considered pulsed motion of the piston such that

$$f(+\infty) = \text{const}, \; f'(t_1)t_1 \to 0, \; f''(t_1) \to 0 \text{ при } t_1 \to \infty \qquad (3.8)$$

then after a fairly long time (for $t_1 \to \infty$) the temperature of the piston surface reaches the value

$$T(\infty, 0) = -\tau^2 f(+\infty) \qquad (3.9)$$

This means that behind the spreading acoustic compression pulse ($f(+\infty) > 0$) the temperature is lowered, while behind a rarefaction pulse ($f(+\infty) < 0$) it is increased

If condition (3.8) is not satisfied, the perturbation of gas temperature indefinitely increases some time after the passing of the wave. This means that for such piston motions the flow is not quasi- isentropic throughout the region when time t_1 is fairly long. We restrict our considerations to acoustic pulses only.

In the distant zone $t_2 \sim 1$, $x_2 \sim 1$ (we recall that now in (1.11) we have $a_0 = a_{T0}$) the assumptions (1.7) and (3.1) also yield formulas (3.4) and the integrals [2]

$$u = \rho = p = -s = q / (16 \tau_0) \qquad (3.10)$$

The derivative of the radiant energy flux in the one-term approximation (2.7) is related to temperature perturbation by Eqs. (2.5) and (2.8). The lacking equation can be derived from (1.5) [2]. After transformation we obtain the following system of equations

$$\frac{\partial u}{\partial t_2} + u\frac{\partial u}{\partial x_2} = \left(\frac{1}{N_T} + \frac{b_T}{\tau}\right)\frac{\partial^2 u}{\partial x_2^2} - \tau b_T u, \quad w = -\tau^2 \int_{x_2}^{\infty} u(t_2, \xi)\, d\xi \qquad (3.11)$$

$$T = w - \frac{\partial u}{\partial x_2}, \quad N_T = 2\varepsilon N_{\mathrm{Re}} m_{T0}, \quad b_T = \frac{N_{\mathrm{Bo}} n}{32\varepsilon m_{T0}} \left(\frac{\partial \ln \rho}{\partial \ln T_0}\right)_p^2$$

$$m_{T0} = \frac{1}{2\rho_0{}^3 a_{T0}^2}\left(\frac{\partial^2 p}{\partial V_0{}^2}\right)_T, \quad V = \frac{1}{\rho}, \quad \Delta = \varepsilon m_{T0}$$

in which parameter N_T describes the effect of viscosity and parameter b_T which is proportional to the Boltzmann number, defines the effect of radiation. The system of Eqs. (3.11) defines in the one-term exponential approximation the nonlinear propagation of acoustic pulses in a radiative gas when $N_{Bo} \ll 1$.

The initial condition for (3.11) is obtained by joining it with solution (3.3) in the intermediate range (2.10), and is of the form (2.11).

It is not difficult to determine the behavior of gas temperature when $x_2 = -\infty$ Since then $u \to 0$ and $\partial u / \partial x_2 \to 0$ hence

$$T(t_2, -\infty) = w(t_2, -\infty) = -\tau^2 \int_{-\infty}^{+\infty} u(t_2, \xi)\, d\xi$$

Now, integrating the equation for u in (3.11) with respect to the coordinate within the limits $-\infty$ to $+\infty$, and solving the obtained differential equation with allowance for (3.9), we obtain

$$T(t_2, -\infty) = -\tau^2 f(+\infty) \exp(-\tau^2 b_T t_2) \tag{3.12}$$

Formulas (3.7) and (3.12) completely define the behavior of temperature at the piston surface : first, the perturbation of the temperature absolute value increases and, then is damped in conformity with the exponential law.

The damping of a triangular compression pulse computed for $\tau = 1$ in an inviscid gas ($N_T = \infty$) is shown in Fig. 3. If parameter b_T is not much different from unity (Fig. 3, a for $b_T = 1$ and Fig. 3, b for $b_T = 0.1$), the perturbations are very quickly damped. When parameter b_T is very small (Fig. 3, c for $b_T = 0.01$) a narrow zone with a sharp velocity gradient, which propagates at a velocity somewhat higher than the isothermal speed of sound, appears in the stream. It follows from the equation for T in (3.11) that the temperature perturbation sharply increases (see Fig. 3, d where the perturbations of velocity and temperature are shown by dash and solid lines, respectively).

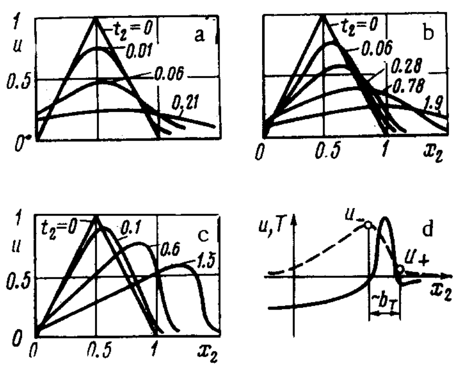

Fig. 3

By expanding velocity u in an asymptotic series in integral powers of parameter b_T we obtain from (3.11) the first approximation equation

$$\frac{\partial u}{\partial t_2} + u\frac{\partial u}{\partial x_2} = 0 \tag{3.13}$$

The solution of this equation for fairly long times t_2 may contain discontinuities, even if at the initial instant velocity u is continuous. Let t_{20} and x_{20} be, respectively, the instant and the coordinate of incipient discontinuity, $u_-(t_2)$ and $u_+(t_2)$ be the velocities of gas immediately behind and in front of the discontinuity, and $c_g(t_2)$

be the translational velocity of the latter. To reveal the real structure of such discontinuity we introduce the inner variable

$$z = \frac{\tau}{b_T}\left[x_2 - x_{20} - \int_{t_{20}}^{t_2} c_S(\xi)\, d\xi\right]$$

and again use the expansion in integral powers of parameter b_T . Then from (3. 11) we have the following internal equation :

$$[u - c_S(t_2)] \frac{\partial u}{\partial x_2} = \frac{\partial^2 u}{\partial x_2^2} \tag{3.14}$$

Asymptotic joining with the external solution defined by Eq. (3. 13) provides the required solution of Eq. (3. 14) and makes it possible to determine velocity $c_S(t_2)$

$$u(t_2, z) = c_S(t_2) - \frac{\Delta_S(t_2)}{2} \operatorname{th}[\Delta_S(t_2) z], \quad c_S = \frac{u_- + u_+}{2}, \quad \Delta_S = u_- - u_+$$

The obtained solution is of the same form as the Taylor solution for the structure of a viscous compression shock. It makes it possible to determinate from (3. 11) the temperature peak shape in the zone of sharp velocity gradients

$$T(t_2, z) = w_S(t_2) + \frac{\tau \Delta_S^2}{2b_T} [\operatorname{ch}(\Delta_S z)]^{-2}$$

where $w_S(t_2)$ is the perturbation of radiant energy density at the discontinuity of the external solution defined by Eq. (3. 13). We have

$$w_S(t_2) = -\tau^2 \int_{x_S(t_2)}^{\infty} u(t_2, x_2)\, dx_2, \qquad x_S(t_2) = x_{20} + \int_{t_{20}}^{t_2} c_S(\xi)\, d\xi$$

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DIFFRACTION OF A SHOCK WAVE ON A THIN WEDGE MOVING AT SUPERSONIC SPEED UNDER THE CONDITIONS OF SPORADIC WAVE INTERACTION

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The subject of present investigation is the diffraction of a shock wave of arbitrary intensity on a thin wedge moving at a supersonic speed. The plane of the shock wave forms an almost right angle with the symmetry plane of the wedge. The interaction between the fronts is assumed sporadic. Studying the pressure perturbation along the front, a singularity of the type similar to that appearing when a weak pressure jump is diffracted on a wedge of finite opening angle with an attached shock, is discovered. This case was dealt with in [1]. The boundary value problem which is solved here using the results of [2, 3] enables us to find the pressure perturbations at the wall and along the shock front, and obtain the expression for the front in terms of elementary functions. The above problem was analyzed for the case of regular interaction in [3], where a method of generalizing the solution to the case of sporadic interaction was also suggested. The method however turned out to be impracticable.

1. A thin wedge moves through a quiescent perfect gas at a supersonic speed $a_\infty M_\infty$ where a_∞ denotes the speed of sound in gas. The half apex angle of the wedge ε is a small parameter of the problem. At the instant $t = 0$ the edge of the wedge encounters the front of the plane shock wave of arbitrary intensity propagating at the speed $a_\infty M$ The plane of the shock wave forms an angle $\chi = \pi / 2 - \delta$, which is nearly a right angle, with the plane of symmetry of the wedge (angle δ is of the order of ε).

The self-similar plane motion arising at $t > 0$ represents a perturbation in a homogeneous flow behind the shock wave.